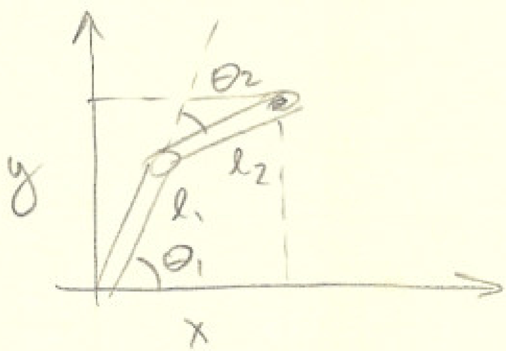
Inverse Kinematics

Consists of the determination of the joint variables coorispording to a given end effector possition and orientation.

The inverse kinematics problem is in general more complex than the forward kinematics problem for the following reasons.

* The equations to solve are in general non-linear, thus it is not always possible to find closed form solution.
* Multiple solutions may exist
* Infinite solutions may exist.
* There may not be admissible solutions.

EX



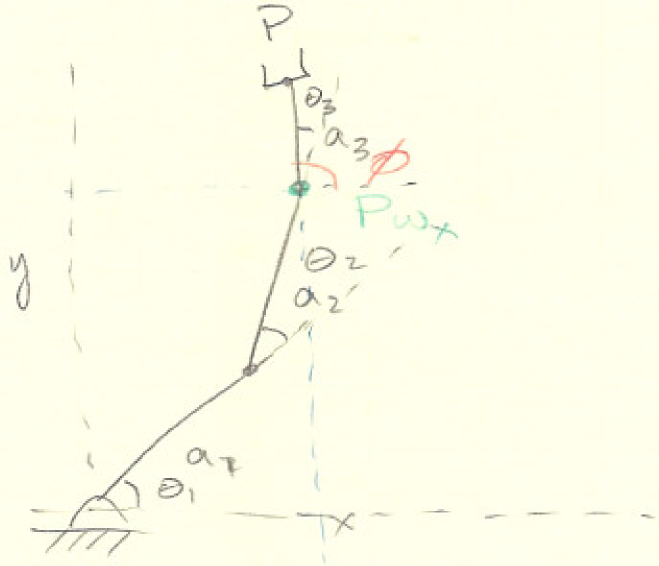
$$x = l_1\cos\theta_1 + l_2\cos(\theta_1-\theta_2)$$
$$y = l_1\sin\theta_1 + l_2\sin(\theta_1-\theta_2)$$

Now given x, y find θ_1 & θ_2

This is the inverse kinematics.

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \rightarrow \boxed{\begin{matrix}\text{Inv}\\\text{Kin}\end{matrix}} \rightarrow \begin{pmatrix}\theta_1\\\theta_2\\\theta_3\end{pmatrix}$$

Ex 3 link manipulator (planar)



P is @ P_x, P_y

ϕ is the angle of the end effector.

$$\theta_1 + \theta_2 + \theta_3 = \phi \qquad (1)$$

$$P_{wx} = P_x - a_3 \cos\phi$$

$$P_{wy} = P_y - a_3 \sin\phi$$

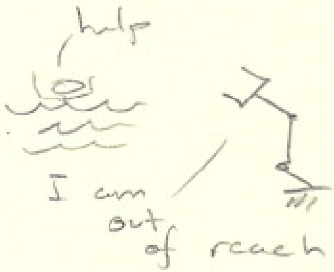
$$P_{wx} = a_1 C_{\theta 1} + a_2 C_{12} \qquad (2)$$

$$P_{wy} = a_1 S_1 + a_2 S_{12} \qquad (3)$$

Squaring and summing (2) & (3)

$$P_{wy}^2 + P_{wx}^2 = a_1^2 + a_2^2 + 2a_1a_2C_2$$

$$\therefore \quad C_2 = \frac{P_{wx}^2 + P_{wy}^2 - a_1^2 - a_2^2}{2a_1a_2}$$



if this is not $\in(-1,1)$ it is outside the range of the robot.

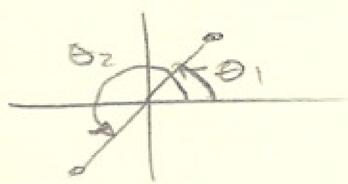
The existence condition

$$-1 \le C_2 \le 1$$

$$-1 \le \frac{P_{wx}^2 + P_{wy}^2 - a_1^2 - a_2^2}{2a_1a_2} \le 1$$

$$S_2 = \pm\sqrt{1 - C_2^2}$$

We have to be careful that we do not forget about reverse quadrant solutions of $\tan^{-1}$



having θ_2, the angle θ_1 can be found by substituting θ_2 in ② & ③

$$S_1 = \frac{(a_1 + a_2C_2)P_{wy} - a_2S_2P_{wx}}{P_{wx}^2 + P_{wy}^2}$$

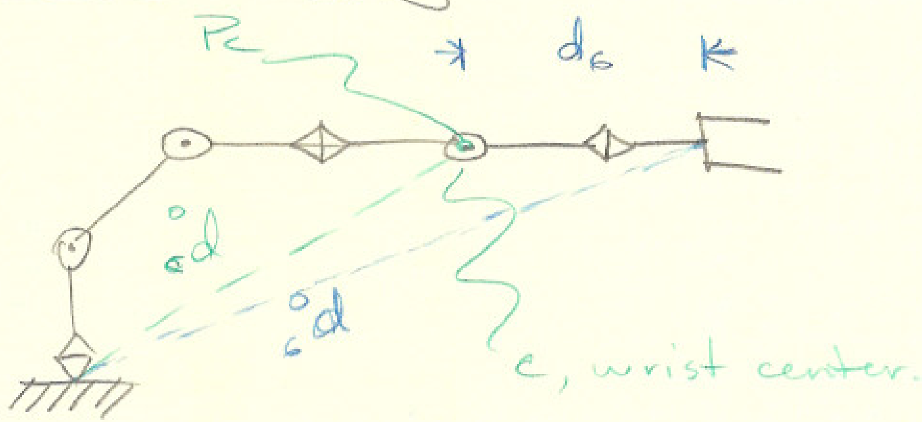
$$C_1 = \frac{(a_1 + a_2 C_2) P_{wx} + a_2 S_2 P_{wy}}{P_{wx}^2 + P_{wy}^2}$$

We will end up with 2 ~~sets~~ of solutions.

Note atan2 (x, y) computes $\tan^{-1}\left(\frac{x}{y}\right)$ but uses the signs of x & y to find which quandrat the value lies in.

Kinematic decoupling

Ex



Given $\quad {}^0_6 H = \left[\begin{array}{c|c} {}^0_6 R & {}^0_6 d \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$

Find the joint variable.

$$P_c = {}^0_6 d - {}^0_6 R \begin{pmatrix} 0 \\ 0 \\ d_6 \end{pmatrix}$$

Now we have eliminated the spherical wrist let $$P_c = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}$$

and $$d = \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix}$$

$$\therefore \quad \begin{aligned} P_x &= d_x - d_6 r_{13} \\ P_y &= d_y - d_6 r_{23} \\ P_z &= d_z - d_6 r_{33} \end{aligned}$$

Using P_c we can find the first 3 variables (q_1, q_2, q_3)

Knowing q_1, q_2, q_3 we can find

$${}^0_3R$$

And then using the fact that

$$R = {}^0_3R \, {}^3_6R$$

$${}^3_6R = {}^0_3R^T R$$

Finally we use ${}^3_6R$ to solve the i/v kinematics for the spherical wrist.